A Producer-Consumer View of Entanglement Purification in Quantum Networks

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***Abstract*— Quantum networks are the basis for a majority of applications in distributed quantum systems and quantum security, and their communication is strongly based on entangled states. The decoherence process causes the fidelity of the entanglements to decay over time, making the network unstable and removing past connections between nodes. To avoid this problem, *purification* algorithms have been developed to aggregate low-fidelity entanglements into a higher-fidelity one. We model the purification process in quantum networks through the lens of a producer-consumer view. To do so, we developed a discrete-time simulation framework based on the QuantumSavory library.**

***Keywords*— quantum networks, entanglement, producer-consumer, purification.**

I. Introduction

Quantum networks are key components for the advancement of distributed quantum computing and trustworthy communication. Communication on these networks relies on entangled states, whose fidelity directly affects the viability of the network. Due to decoherence and other external factors, the fidelity of entangled pairs decays over time. This decay motivates the periodic generation of high-fidelity entangled pairs to mitigate this loss.

Methods such as *purification* propose to sacrifice low-fidelity entanglements to produce higher-fidelity ones, seeking to maintain a higher fidelity over time [1]–[3]. However, it turns out that the purification process may make the network unavailable, raising the question of at what rate purification should be performed.

In this work, we aim to model the purification process in a quantum network through the lens of a producer-consumer system. In this case, the entanglement generation acts as the producer of auxiliary resources, while the consumer process controls the actions such as when to purify or when to consume an entangled pair. We organize our study around two questions:

RQ1: *How do system parameters influence the joint evolution of entanglement fidelity and availability in a concurrent quantum network?*

RQ2: *Which modeling components are needed to turn purification and regeneration into a meaningful fidelity-availability-cost trade-off?*

To address these questions, we developed a discrete-time simulation framework based on the QuantumSavory library [4]. We aim to identify variables that affect how the fidelity of the entanglements evolves over time. It is important to be aware of them in order to properly answer the second question, as a way to guide the purification scheduling and maintain a low rate of unavailability in the network.

At this stage, our goal is focused on validating the modeling framework and understanding its qualitative behavior. In particular, our preliminary results reveal a counterintuitive effect: increasing the demand rate can improve the observed average fidelity. This phenomenon arises because entanglement consumption triggers the regeneration of Bell pairs with maximal fidelity, while the cost of generating such pairs is not yet explicitly modeled.

II. Producer-Consumer Framework

The simulated network consists of nodes that store registers, each of which has two qubits. In our examples, we consider a simple network with only two connected nodes. We decompose the system into two concurrent processes: an auxiliary entanglement generator (producer) and a purification/consumer controller (consumer). They are shown in Algorithms 1 and 2, respectively. They interact through shared registers (slots 1 and 2) and are synchronized via a lock to ensure consistency. Based on such decomposition, the entanglement generation corresponds to resource production and purification/consumption corresponds to resource allocation.

The simulation includes parameters for simulation time, usage and production control of entanglements, and scheduling decisions. In the consumer algorithm, the parameter p defines the probability that the main entangled pair in slot 1 is consumed, simulating the network demand. In the producer algorithm, the parameter q defines the probability an entanglement will be generated in the auxiliary slot, i.e., in slot 2.

It is important to note that both processes operate in different timescales: auxiliary entanglements production occurs in intervals of Δ_2 (Algorithm 1, line 2), while the control loop executes at timestep Δ_1. The purification process is straightforward (Algorithm 2, line 5): if both slots are entangled, a purification process starts, using the entangled pair in the second slot to increase the fidelity of the first one. In case of failure, both slots are reinitialized and a Bell pair is added to the first one.

III. Preliminary Insights

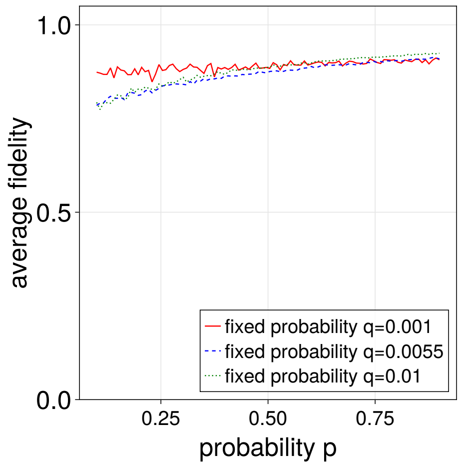
Figure 1a shows the effect of fixing probability q while varying the probability p. Note that the average fidelity increases with demand. This behavior is due to the fact that we generate a Bell pair, which has maximal fidelity, every time an entangled pair is consumed. As we are not taking into account the unavailable time, the generation of such Bell pairs and the purification process are considered instantaneous.

Algorithm 1: AUXILIARY PAIR PRODUCER

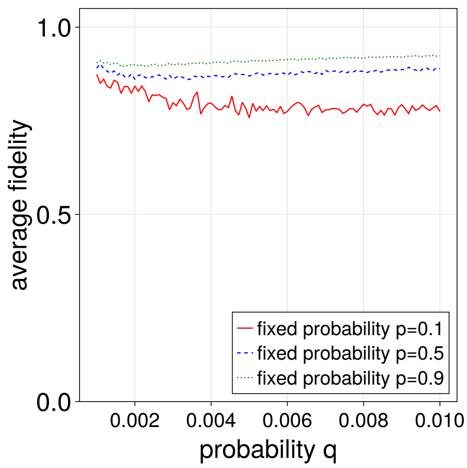
```
Input: Attempt interval Δ_2, success probability q
Shared: Slot 2 registers, lock
while simulation is running do
    Wait Δ_2;
    Acquire lock on slot 2;
    Draw v ~ Bernoulli(q);
    if v = 1 then
        Create Bell pair in slot 2;
        Mark slot 2 as entangled;
    Release lock on slot 2;
```

Algorithm 2: PURIFICATION AND CONSUMPTION

```
Input: Timestep Δ_1, demand probability p
Shared: Slot 2 registers, lock
while simulation is running do
    Wait Δ_1;
    Acquire locks on slot 1 and slot 2;
    // Purification has priority
    if slot 1 and slot 2 are entangled then
        Attempt purification;
        if success then
            Keep result in slot 1;
            Clear slot 2;
        else
            Reset system;
            Reinitialize slot 1;
            Clear slot 2;
    // Consumption
    if slot 1 is entangled then
        Draw u ~ Bernoulli(p);
        if u = 1 then
            Estimate fidelity from event log;
            Consume slot 1;
    // Ensure availability
    if slot 1 is empty then
        Create Bell pair in slot 1;
    Release locks;
```



(a) Average fidelity as a function of p. The average fidelity increases when p increases due to the reset process that occurs in the network.



(b) Average fidelity as a function of q. Interestingly, the fidelity decreases when q increases.

Fig. 1: Average fidelity as a function of the demand probability p and the production probability q, respectively.

Figure 1b shows that, fixing p, increasing q decreases fidelity. One of our goals is to understand such behavior. Also, as we have mentioned, the unavailable time is not taken into account, and therefore we can always reset the network aiming to obtain a maximal fidelity state. Ensuring that such actions are penalized during simulation would result in more realistic results and we expect this to clarify correlations between unavailability and a good purification scheduling.

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AI usage. AI was used to assist with LaTeX formatting and language editing.

Replication Package. We provide a replication package with the simulation code at `https://github.com/Mth0/producer-consumer-quantum-purification`.

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