Frequency Assignment in Mobile Networks via the Bandwidth Coloring Problem: A Hybrid Quantum–Classical Approach

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***Abstract*— Dense mobile networks must reuse radio spectrum while limiting interference among nearby transmitters, a requirement formalized by the Frequency Assignment Problem (FAP). We model the FAP as a Graph Coloring Problem (GCP) and as a Bandwidth Coloring Problem (BCP), where edge weights encode minimum spectral separations. We present a hybrid quantum–classical pipeline for the Noisy Intermediate-Scale Quantum (NISQ) regime: Louvain partitioning decomposes the interference graph into qubit-compatible subgraphs; a VQE-inspired routine with a Reconfigurable-Beam-Splitter (RBS) ansatz colors each subgraph while preserving one-hot feasibility; and a greedy classical stage merges the partial colorings and repairs boundary conflicts. The workflow follows the decomposition, local-optimization, and reconciliation structure of hierarchical hybrid coloring methods, but extends the study to weighted BCP constraints. On small synthetic graphs, the pipeline produced valid final colorings and removed all detected conflicts for both formulations.**

***Keywords*— Quantum computing, frequency assignment problem, bandwidth coloring problem, graph coloring, VQE, NISQ.**

I. Introduction

Dense mobile networks depend on efficient frequency reuse. When neighboring cells, antennas, or wireless links operate on incompatible channels, co-channel or adjacent-channel interference can reduce the quality of service. This engineering task is commonly formalized as the *Frequency Assignment Problem* (FAP), in which channels are assigned to network elements under interference constraints [1], [2]. A standard abstraction maps network elements to vertices, potentially interfering pairs to edges, and available channels to colors, yielding the *Graph Coloring Problem* (GCP). A more expressive version is the *Bandwidth Coloring Problem* (BCP), where edge weights specify minimum spectral separation between adjacent vertices, thereby representing adjacent-channel constraints as well as simple color inequality [3].

This work studies a small-scale hybrid quantum–classical approach for these coloring-based FAP models in the Noisy Intermediate-Scale Quantum (NISQ) setting [4]. The high-level design is based on the hierarchical strategy of Liu *et al.* [5]: decompose the original graph, apply a quantum variational routine to smaller subgraphs, and use classical post-processing to merge the partial assignments and repair conflicts. Our implementation differs in three points. First, decomposition is performed by Louvain community detection. Second, each local subproblem is optimized with a VQE-inspired RBS ansatz that preserves the one-hot coloring constraint by construction. Third, the same encoding is used for both the unweighted GCP and the weighted BCP, allowing minimum frequency separations to be considered in the cost Hamiltonian.

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II. Problem Formulation

Let $G = (V, E)$ be an interference graph and $C = \{0, \ldots, |C| - 1\}$ the available frequency set. A coloring is a map $f : V \to C$. In the GCP an edge (u, v) is satisfied when $f(u) \neq f(v)$. In the BCP, each edge has a positive integer weight $w(u, v)$ and is feasible only when

$$|f(u) - f(v)| \geq w(u, v), \tag{1}$$

so the ordinary GCP is recovered when $w \equiv 1$. Each vertex u is encoded by $|C|$ binary variables $x_{u,c} \in \{0, 1\}$ with exactly one active color,

$$\sum_{c=0}^{|C|-1} x_{u,c} = 1, \qquad \forall u \in V. \tag{2}$$

On qubits, $x_{u,c}$ is represented by the number operator $\hat{n}_{u,c} = \frac{1}{2}(I - Z_{u,c})$ in a vertex-major layout. The GCP Hamiltonian counts monochromatic edges,

$$H_{\mathrm{GCP}} = \sum_{(u,v)\in E} \sum_{c=0}^{|C|-1} \hat{n}_{u,c}\hat{n}_{v,c}, \tag{3}$$

whose zero-energy states are proper colorings. The BCP Hamiltonian penalizes color pairs whose distance is smaller than the required edge bandwidth:

$$\begin{aligned} H_{\mathrm{BCP}} =& \frac{1}{2} \sum_{(u,v)\in E} \sum_{c=0}^{|C|-1} \sum_{d=0}^{w(u,v)-1} (w(u,v) - d) \\ & \times \left(\hat{n}_{u,c}\hat{n}_{v,c+d} + \hat{n}_{u,c}\hat{n}_{v,c-d}\right), \end{aligned} \tag{4}$$

where invalid color indices are omitted and the $d = 0$ term is counted only once. If $w \equiv 1$, only equal-color violations remain and Eq. (4) collapses to Eq. (3).

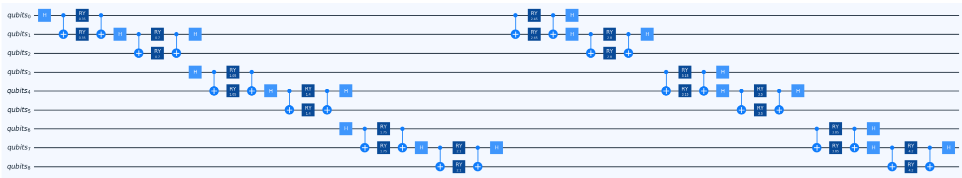

Fig. 1: RBS-based variational ansatz for one subgraph ($n = |C| = 3$; 9 qubits, two layers). RBS gates, decomposed into elementary gates, mix a single excitation within each vertex block and preserve one-hot feasibility.

III. Methodology

The proposed workflow maps the FAP instance to a coloring problem, decomposes the graph, solves each subgraph variationally, and then reconciles the local solutions. The decomposition stage uses the Louvain algorithm [6], a modularity-based community-detection heuristic. A community is accepted only if $n_{\text{sub}}|C| < 20$ qubits, where n_{sub} is its number of vertices; otherwise, it is recursively re-partitioned or split until the budget is met. This step is motivated by NISQ limits and by the hierarchical pipeline of Ref. [5].

Each subgraph is colored by a VQE-inspired routine [7] using *Reconfigurable Beam Splitter* (RBS) gates [8]. An RBS gate is a two-qubit Givens rotation on the single-excitation subspace,

$$\text{RBS}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

Each vertex block of $|C|$ qubits is initialized as $|10\cdots0\rangle$ by one X gate and then mixed by RBS layers applied only within that block (Fig. 1). Because RBS gates conserve excitation number, every basis state in the support has exactly one active qubit per vertex and therefore satisfies Eq. (2). The variational search is restricted to the feasible one-hot subspace and does not require an additional penalty term for one-hot validity. The parameters $\vec{\theta}$ minimize $\langle\psi(\vec{\theta})|H|\psi(\vec{\theta})\rangle$ with the derivative-free COBYLA optimizer [9]; after optimization, the final state is sampled and the most frequent bitstring is decoded as the local coloring.

The subgraph colorings are concatenated into a global assignment. Since edges crossing different communities are not optimized during local VQE execution, recombination may introduce conflicts: $f(u) = f(v)$ for the GCP or $|f(u) - f(v)| < w(u, v)$ for the BCP. The reconciliation stage scans the conflicting vertices and recolors each one with the smallest color compatible with its already assigned neighbors. This greedy repair is intentionally simple; it serves as a lightweight classical merge step rather than as a full global optimizer.

IV. Results

Experiments were run on a noiseless state-vector simulator using NetworkX to generate an Erdős–Rényi graph $G(n = 10, p = 0.5)$ with a fixed seed. The instance has 21 edges and average degree 4.2; for the BCP, integer edge weights were sampled from $\{1, 2\}$. Two rounds were evaluated: an unweighted GCP with four colors and a weighted BCP with five colors. In both cases, the local variational stage generated assignments that contained boundary conflicts after recombination, and the greedy reconciliation stage removed all detected violations. The BCP required more runtime because it used a larger color set and because Eq. (4) adds terms for near-color violations, not only equal colors.

TABLE I: Results for the unweighted GCP and weighted BCP instances.

Round	**Vert.**	**Colors**	**Confl. before**	**Confl. after**	**Time (s)**
Simple GCP	10	4	3	0	≈ 5.98
Weighted BCP	10	5	4	0	≈ 8.94

V. Discussion and Conclusion

The results indicate that the full hybrid flow works end to end on small instances: FAP-to-coloring modeling, Louvain decomposition, local RBS-based variational coloring, and classical reconciliation. They should not be read as evidence of quantum advantage or as a competitive benchmark against classical FAP solvers. The main limitations are the synthetic graph model, the absence of propagation, power, mobility, and demand constraints, the noiseless simulator, the small instance size, and the local nature of the greedy repair. Future work will compare against classical heuristics and integer/constraint programming, test larger and more realistic network instances, execute noisy simulations and hardware runs, and evaluate a constraint-preserving QAOA variant with an XY mixer [10], [11] initialized in a one-excitation state per vertex block.

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